

Learning Curve

12

SLOB Mapped against the Module

To equip oneself with application-oriented knowledge of Learning Curve to facilitate management decisions for optimisation through resource allocation, managing competition, work scheduling and managing cost overrun, demand estimation, production and cost analysis etc.

Module Learning Objectives

After studying this module, the students will be able to:

- ⦿ Understand the phenomenon of Learning and visualize the associated Curve.
- ⦿ Understand how the percentage learning rate applies to the doubling of output.
- ⦿ Use Learning Curve theory successfully for Pricing decisions, Work scheduling and setting of Standards.
- ⦿ Identify the situations where Learning effect should be incorporated in Industry.

Learning is the process by which human beings acquire a vast variety of competencies, skills, knowledge and attitudes. In any environment if a person is assigned to do the same task again and again then after a period of time, there is an improvement in his performance. This happens due to the fact that the phenomenon of Learning takes place causing increase in competency of the person carrying out the task. Repetition of the task is likely to make the person more confident and knowledgeable and will eventually result in a more efficient and rapid operation. Ultimately the learning process will stop after continually repeating the job. As a consequence, the time to complete a task will initially decline and then stabilize once efficient working is achieved. The cumulative average time per unit is assumed to decrease by a constant percentage every time the output doubles. Cumulative average time refers to the average time per unit for all units produced so far, from and including the first one made.

When data of performance of such a repeated task are collected over a period of time and plotted on a graph paper we get a curve called Learning Curve. A Learning Curve is actually a correlation between a learner's performance on a task and the number of attempts or time required to complete the task. The curve shows that if a task is performed over and over then less time is required at each iteration. Thus, Learning Curve theory proposes that a learner's efficiency in a task improves over time the more the learner performs the task.

Though serious studies about the effect of learning on a repetitive task began in the twentieth century, the similar concept was first coming into the surface as early as 1885 when the German psychologist Dr. Hermann Ebbinghaus first conducted experimental research on human memory. He was interested in discovering why new information learnt by people tends to fade away from the memory over a period of time. The data collected by him was plotted on a graph to get a curve which was named by him as the Forgetting curve. He also discovered, if learning is rehearsed and repeated at regular intervals, people forget less.

Significance of Learning Curve

Learning Curve finds a very important role in Cost Analysis, Cost Estimation and Efficiency studies. Cost predictions especially those relating to direct labour cost must allow for the effect of learning process. This technique is a mathematical technique. It can be very much used to accurately and graphically predict cost. It is a geometrical progression, which reveals that there is steadily decreasing cost for the accomplishment of a given repetitive operation. As the identical operation is increasingly repeated, the amount of decrease is less and less with each successive unit produced. The slope of the curve can be expressed as a percentage. Experience curve, Improvement curve and Progress curve are other terms which can be synonymously used. Learning Curve is essentially a measure of the experience gained in production of an article by an individual or organisation. As more units are produced, people involved in production become more efficient than before. Each subsequent unit takes fewer man-hours to produce. The amount of improvement will differ with each type of article produced. This improvement or experience gained is reflected in decrease in man-hours or cost.

Phases in Learning Curve

The Learning Curve passes through three different phases. In the first phase, there will be gradual increase in production rate until the maximum expected rate is reached and this phase is generally steep. In the second phase, the learning rate will gradually deteriorate because of the limitations of equipment. In the third phase, the production rate begins to decrease due to a reduction in customer requirements and increase in costs.

Learning Curve Ratio

Under the Learning Curve model, the cumulative average time per unit produced is assumed to fall by a constant percentage every time total output of the unit doubles. Learning Curve is a geometrical progression which reveals that there is a steady decrease in cost as the identical operation is increasingly repeated.

Learning Curve is essentially a measure of the experience gained in production of an article by an organisation. As more and more units are produced, workers involved in production become more efficient than before. Each subsequent unit takes fewer man-hours to produce. The Learning Curve exists during a worker's startup or familiarization period on a particular job. After the limits of experimental learning are reached, productivity tends to stabilize and no further improvement is possible. The Learning Curve ratio or Learning Ratio can be calculated with the help of the following formula:

$$\text{Learning Curve ratio} = \frac{\text{Average labour cost of first } 2n \text{ units}}{\text{Average labour cost of first } n \text{ units}}$$

If the Average labour cost of the first 50 units of a Product is ₹ 40 and that of the first 100 units is ₹ 32 then the Learning Ratio is calculated as $32/40 = 0.8$ or 80% which means that every time the output doubles, the average labour cost is reduced to 80% of the previous amount. In this case, average labour cost of the first 200 units will be 80% of that of the first 100 units = 80% of ₹32 = ₹25.60. Conventionally Learning Curves are referred to by complements of their decrease rates. In the above example 80% Learning Curve showed a 20% decrease in average labour cost for every doubling of the output. Learning percentage is actually the Slope of the Learning Curve. It can be noted that a 100% Learning implies no decrease of average labour cost (which means "No Learning" effect).

Example 1

An operation to produce an item is known to have an 80% Learning Curve. It has taken a worker 20 hours to complete the operation for the first unit. Determine the expected completion times for the 2nd, 4th and 8th units.

Solution:

As per the Theory of Learning Curve, each time the cumulative output doubles, the time per unit should be equal to the previous time multiplied by the learning percentage (80% in this case)

Unit	1st	2nd	4th	8th
Expected Completion time (hours)	20	$20 \times 0.8 = 16$	$20 \times (0.8)^2 = 12.8$	$20 \times (0.8)^3 = 10.24$

Here the important point to remember is the fact that the time reduction per unit becomes smaller and smaller as

the number units produced is increased. The reduction of time from the 1st unit to the 2nd is 4 hours whereas that from the 4th to 8th is only 2.56 hours. Also the reduction of time follows a Geometrical Progression with Common Ratio = Learning ratio

Thus, when the units follow a doubling pattern then the time as well as cost can be easily calculated as shown above. But when they do not follow the doubling pattern (i.e. for the 3rd, 5th, 6th etc. units) then we have to use Wright's formula (discussed afterwards) to find the time.

Areas of Consequence

- (i) A Standard Costing system would need to set standard labour times after the Learning Curve had reached a plateau.
- (ii) A budget will need to incorporate a learning cost factor until the plateau is reached.
- (iii) A budgetary control system incorporating labour variances will have to make allowances for the anticipated time changes.
- (iv) Identification of the Learning Curve will permit the company to better plan its marketing, work scheduling, recruitment and material acquisition activities.
- (v) The decline in labour costs will have to be considered when estimating the overhead apportionment rate.
- (vi) As the employees gain experience they are more likely to reduce material wastage.

Graphical Presentation of Learning Curve

The Learning Curve (not to be confused with experience curve) is a graphical representation of the phenomenon explained by Theodore P. Wright in his "Factors Affecting the Cost of Airplanes", 1936. It refers to the effect that learning had on labour productivity in the aircraft industry, which translates into a relation between the cumulative number of units produced (X) and the average time (or labour cost) per unit (Y), which resulted in a convex downward slope, as seen in the diagram below.

There is a simple rationalization behind all this: the more units produced by a given worker, the less time this same worker will need to produce the following units, because he will learn how to do it faster and better. Therefore when a firm has higher cumulative volume of production, its time (or labour cost) per unit will be lower.

Wright's Learning Curve model is defined by the following function:

$$Y = aX^b$$

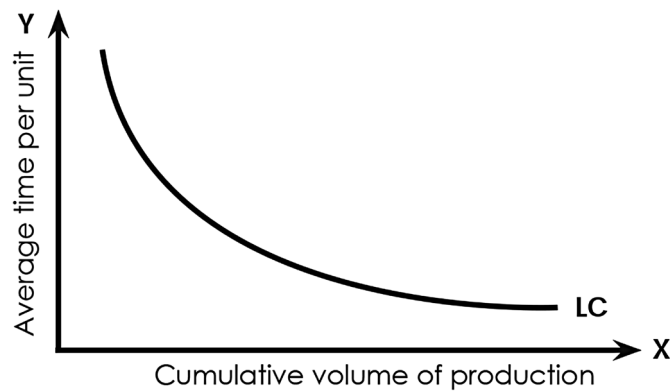
where:-

Y = average time (or labour cost) per unit

a = time (or labour cost) for the 1st unit

X = cumulative volume of production

b = learning index = $[\log (\text{learning percentage}) / \log 2]$



Most important implication arising from the curve is the fact that when the average time or labour cost per unit decreases with the increase in cumulative output then the firm which is producing more and for a longer period will dominate the market because of its lower average time or labour cost.

Pros and Cons of the Learning Curve Theory

Pros of the Learning Curve Theory -

Using a Learning Curve will help a business to improve the performance and productivity of their workforce and reduce costs. When used to track or predict performance, it can provide psychological motivation and also facilitate in strategic planning.

- ⦿ Performance improvement can't happen on its own and is connected with learning. By incorporating a learning culture within the organisation where employees are encouraged and supported to keep learning, performance levels can be expected to increase.
- ⦿ The rate of learning is also considered to be consistent enough that trends can be established using the Learning Curve, enabling better forecasting and business decisions.

Cons of the Learning Curve Theory –

A Learning Curve is dependent on the assumptions made about performance. Many variables can impact learning and future performance.

- ⦿ Using a Learning Curve to predict overall performance of larger groups or processes means that many assumptions are made on variables like motivation, sociological factors, workplace dynamics, training resources and previous knowledge or experience.
- ⦿ A Learning Curve that may not show expected results would need further analysis to determine the underlying variables impacting its shape, as the curve does not tell the whole story.

Uses of Learning Curve

Learning Curve is now being widely used in business. Some of the uses are as follows:

1. Where applicable the Learning Curve suggests great opportunities for cost reduction to be achieved by improving learning.
2. The Learning Curve concept suggests a basis for correct staffing in continuously expanding production. The curve shows that the work force need not be increased at the same rate as the prospective output. This also helps in proper production planning through proper scheduling of work; providing manpower at the right moment permitting more accurate forecast of delivery dates.
3. Learning Curve concept provides a means of evaluating the effectiveness of training programs. What level of cumulative cost reduction do they accomplish? How does the Learning Curve for this group or shop compare with others? Whether any of the employees who lack the aptitude to meet normal Learning Curve should be eliminated.
4. Learning Curve is frequently used in conjunction with establishing bid prices for contracts. Usually, the bid price is based on the cumulative average unit cost for all the units to be produced for a given contract. If production is not interrupted, additional units beyond this quantity should be considered at the increment costs incurred, and not at the previous cumulative average. If the contract agreement so provides, a contract may be cancelled and production stopped before the expected efficiency is reached. This would mean that the company having quoted on the basis of cumulative average unit cost is at a disadvantage because it cannot reap the benefit of Learning. The contractor must provide for these contingencies so that it will be reimbursed for such loss.
5. The use of Learning Curve, where applicable, is important in finding the working capital required. If the requirement is based on average cumulative unit cost, the revenues from the first few units may not cover the actual expenditures. For instance, if the price was based on the average cumulative unit time of 3.28 hours the first unit when produced and sold will cause a deficit of 4.72 hours ($8.00 - 3.28$). Provision should therefore be made to cover the deficit of working capital in the initial stages of production.
6. As employees become more efficient, the rate of production increases and so more materials are needed, the work-in-progress inventory turns over faster, and finished goods inventory grows at an accelerated rate. A knowledge of the Learning Curve assists in planning the inventories of materials, work-in-progress, and finished goods.
7. Learning Curve techniques are useful in exercising control, Variable norms can be established for each situation, and a comparison between these norms and actual expenses can be made. Specific or average incremental unit cost should be used for this purpose.
8. The Learning Curve may be used for make-or-buy decisions especially if the outside manufacturer has reached the maximum on the Learning Curve help to calculate the sensitive rates in wage bargaining.

Limitations to the usefulness of the Learning Curve

The following points limiting the usefulness of Learning Curves should be noted:-

1. The Learning Curve is useful only for new operations where machines do not constitute a major part of the production process. It is not applicable to all production situations. E.g. new and experienced workmen.
2. The Learning Curve assumes that the production will continue without any major interruptions. If for any reason the work is interrupted, the curve may be deflected or assume a new slope.
3. Changes other than learning may affect the Learning Curve. For example, improvement in facilities,

arrangements, and equipment as well as personnel morale and performance may be the factors influencing the curve. On the other hand, negative developments in employee attitudes may also affect the curve and reverse or retard the progress of improvement.

4. The characteristics for 80 percent Learning Curve as originally obtained in the airline industry in U.S.A. has been usually accepted as the percentage applicable to all industries. Studies show that there cannot be a unique percentage which can be universally applied.

Factors affecting Learning Curve

1. While pricing for bids, general tendency is to set up a very high initial labour cost so as to show a high Learning Curve. This makes the Learning Curve useless and misleading.
2. The method of production, i.e. whether it is labour oriented or machine oriented influences the slope of the learning.
3. When labour turnover rate is high, management has to train new workers frequently. In such situations the company may never reach its maximum efficiency potential. One of the important requisites of the Learning Curve concept is that there should be uninterrupted flow of work. The fewer the interruptions, the greater will be the improvement in efficiency.
4. Changes in a product or in the methods of production, designs, machinery, or the tools used affect the slope of the Learning Curve. All these have the effect of starting learning afresh because of new conditions. If the changes are frequent, there may be no learning at all.
5. Also other factors influencing the Learning Curve are labour strikes, lock outs and shutdowns due to other causes that also affect the Learning Curve. In each such case there is interruption in the progress of learning.

As far as possible the effects of above factors should be carefully separated from the data used to establish the curve. The effects of these factors must also be separated from the actual costs used to measure the performance. Unless this is done analysis of the projected cost or the actual cost will not be meaningful.

The Experience Curve

The more experience a firm has in producing a particular product, the lower is its cost.

The experience curve is an idea developed by the Boston Consulting Group (BCG) in the mid-1960s. Working with a leading manufacturer of semiconductors, the consultants noticed that the company's unit cost of manufacturing fell by about 25% for each doubling of the volume that it produced. This relationship they called as the experience curve: the more experience a firm has in producing a particular product, the lower is its cost. Bruce Henderson, the founder of BCG, put it as follows:

Costs characteristically decline by 20-30% in real terms each time accumulated experience doubles. This means that when inflation is factored out, costs should always decline. The decline is fast if growth is fast and slow if growth is slow.

There is no fundamental economic law that can predict the existence of the experience curve, even though it has been shown to apply to industries across the world. Its truth has been proven inductively, not deductively, and if it is true in service industries such as investment banking or legal advice, the lower costs are clearly not passed on to customers.

By itself, the curve is not particularly earth-shattering. Even when BCG first expounded the relationship, it had been known since the second world war that it applied to direct labour costs. Less labour was needed for a given

output depending on the experience of that labour. In aircraft production, for instance, labour input decreased by some 10–15% for every doubling of that labour’s experience.

The strategic implications of the experience curve came closer to earth-shattering. For if costs fell (fairly predictably) with experience, and if experience was closely related to market share (as it seemed it must be), then the competitor with the biggest market share was going to have a big cost advantage over its rivals. Being market leader is a valuable asset that a firm relinquishes at its peril.

This was the logical underpinning of the idea of the growth share matrix. The experience curve justified allocating financial resources to those businesses (out of a firm’s portfolio of businesses) that were (or were going to be) market leaders in their particular sectors. This, of course, implied starvation for those businesses that were not and never would be market leaders.

Over time, managers came to find the experience curve too imprecise to help them much with specific business plans. Inconveniently, different products had curves of a different slope and different sources of cost reduction. They did not, for instance, all have the same downward gradient as the semiconductor industry, where BCG had first identified the phenomenon. A study by the Rand Corporation found that “a doubling in the number of [nuclear] reactors [built by an architect–engineer] results in a 5% reduction in both construction time and capital cost”.

Part of the explanation for this discrepancy was that different products provided different opportunities to gain experience. Large products (such as nuclear reactors) are inherently bound to be produced in smaller volumes than small products (such as semiconductors). It is not easy for a firm to double the volume of production of something that takes over five years to build, and whose total market may never be more than a few hundred units.

In theory, the experience curve should make it difficult for new entrants to challenge firms with a substantial market share. In practice, new firms enter old industries all the time, and before long many of them become major players in their markets. This is often because they have found ways of bypassing what might seem like the remorseless inevitability of the curve and its slope. For example, experience can be gained not only first-hand, by actually doing the production and finding out for yourself, but also second-hand, by reading about it and by being trained by people who have first-hand experience. Furthermore, firms can leapfrog over the experience curve by means of innovation and invention. All the experience in the world in making black and white television sets is worthless if everyone wants to buy colour ones.

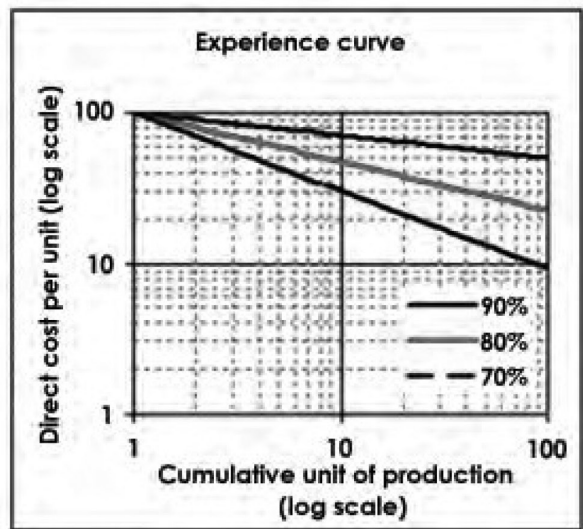
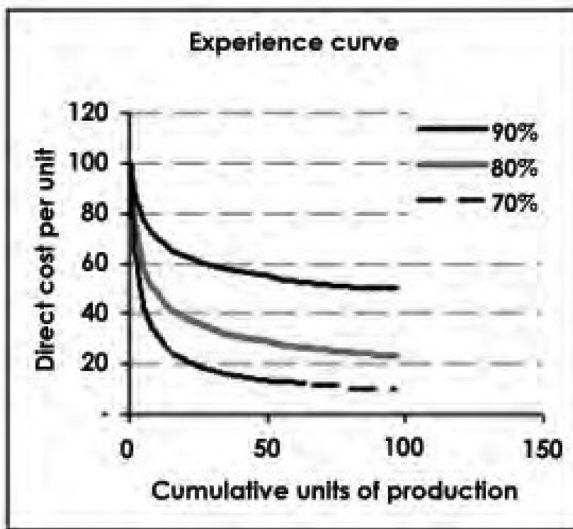


Illustration 1

The usual Learning Curve model is $Y = ax^b$ where

Y is the average time per unit for x units and 'a' is the time for first unit

x is the cumulative number of units

b is the learning coefficient and is equal to $(\log 0.8)/(\log 2) = -0.322$ for a learning rate of 80%

Given that $a = 10$ hours, you are required to Calculate:

- (i) The average time for 20 units.
- (ii) The total time for 30 units.
- (iii) The time for units 31 to 40.

Given that $\log 2 = 0.301$, Antilog of 0.5811 = 3.812

$\log 3 = 0.4771$, Antilog of 0.5244 = 3.345.

$\log 4 = 0.6021$, Antilog of 0.4841 = 3.049.

Solution:

(i) $Y = aX^b$

$$Y = 10(20)^{-0.322}$$

Taking logarithm on both sides

$$\log Y = \log 10 + \log 20^{(-0.322)}$$

$$\log Y = \log 10 - (0.322) \log 20$$

$$= 1 - (0.322) \log 20$$

$$= 1 - (0.322) \times (1.3010)$$

$$= 1 - 0.41892 = 0.5811$$

$$\log Y = 0.5811$$

$$Y = \text{Anti log } (0.5811) = 3.812 \text{ hrs (average time for 20 units)}$$

(ii) $\log Y = \log 10 + \log 30^{(-0.322)}$

$$\log Y = 1 - (0.322) \times (1.4771)$$

$$= 1 - (0.4756) = 0.5244$$

$$Y = \text{anti log } (0.5244) = 3.345 \text{ hrs (average time for 30 units)}$$

$$\text{Total time for 30 units} = 3.345 \times 30 = 100.35 \text{ hrs}$$

(iii) $\log Y = \log 10 + \log 40^{(-0.322)}$

$$= 1 - (0.322) \times (1.6021)$$

$$\log Y = 0.4841$$

$$Y = \text{anti log } (0.4841) = 3.049 \text{ hrs}$$

$$\text{Total time for 40 units} = 40 \times 3.049 = 121.96 \text{ hrs}$$

$$\text{Time from 31 to 40 units} = 121.96 - (100.35) = 21.61 \text{ hrs}$$

Illustration 2

The Learning Curve in management accounting has now become or is going to become an accepted tool in industry, for its applications are almost unlimited. When it is used correctly, it can lead to increased business and higher profits; when used without proper knowledge, it can lead to lost business and bankruptcy. State precisely:

- (i) Your understanding of the Learning Curve;
- (ii) The theory of Learning Curve;
- (iii) The areas where Learning Curves may assist in management accounting; and
- (iv) Illustrate the use of Learning Curves for calculating the expected average unit cost of making—
 - (a) 4 machines (b) 8 machines

Using the data below:

Data:

- Direct Labour needed to make first machine = 1000 hrs.
- Learning Curve = 90%
- Direct Labour cost = ₹ 15 per hour.
- Direct materials cost = ₹ 1,50,000
- Fixed cost for either size orders = ₹ 60,000.

Solution:

Statement showing computation of expected average cost of making 4 machines & 8 machines:

No of machines	Average time (Hours)	Labour cost (@₹ 15/Hr)	Material Cost (₹)	Fixed cost (₹)	Total Cost (₹)
1	1000	15,000	1,50,000	60,000	2,25,000
2	900	13,500	1,50,000	30,000	1,93,500
4	810	12,150	1,50,000	15,000	1,77,150
8	729	10,935	1,50,000	7,500	1,68,435

Average cost of making 4 machines - ₹ 1,77,150

Average cost of making 8 machines - ₹ 1,68,435

Illustration 3

Z.P.L.C experiences difficulty in its budgeting process because it finds it necessary to quantify the learning effect as new products are introduced.

Substantial product changes occur and result in the need for retraining.

An order for 30 units of a new product has been received by Z.P.L.C So far, 14 have been completed; the first unit required 40 direct labour hours and a total of 240 direct labour has been recorded for the 14 units. The production manager expects an 80% learning effect for this type of work.

The company uses standard absorption costing. The costs attributed to the centre in which the unit is manufactured are as follows:

Head	Cost
Direct material	₹ 30.00 per unit.
Direct Labour	₹ 6.00 per hour.
Variable overhead	₹ 0.50 per direct labour hour.
Fixed overhead	₹ 6,000 per 4 week operating period.

There are ten direct employees working a five-day week, eight hours per day. Personal and other downtime allowances account for 25% of total available time.

The company usually quotes a four-week delivery period for orders. You are required to:

Determine whether the assumption of an 80% learning effect is a reasonable one in this case, by using the standard formula $Y = ax^b$

Where Y = the cumulative average direct labour time per unit (productivity)

a = the average labour time per unit for the first batch.

x = the cumulative number of batches produced.

b = the index of learning.

- Calculate the number of direct labour hours likely to be required for an expected second order of 20 units.
- Use the cost data given to produce an estimated product cost for the initial order, examine the problems which may be created for budgeting by the presence of the learning effect.

Use logarithmic tables to find the values of Logarithm and Anti-Logarithm.

Solution:

Total time taken to produce 14 units

$$Y = ax^b$$

$$Y = 40 (14)^{-0.322}$$

$$\begin{aligned} \log Y &= \log 40 - (0.322) \log 14 \\ &= 1.60221 - (0.322) \times 1.1461 \\ &= 1.60221 - 0.3690 = 1.233 \end{aligned}$$

$$Y = \text{Antilog}(1.233) = 17.14$$

$$\begin{aligned} \text{Total time} &= 17.14 \times 14 = 239.96 \\ &= 240 \text{ hours (which is same as the hours recorded)} \end{aligned}$$

So the assumption that learning ratio 80% is reasonable.

- 30 units

$$Y = 40 (30)^{-0.322} = 13.380 \text{ hours (Average time)}$$

50 units

$$Y = 40 (50)^{-0.322} = 11.35 \text{ hours (Average time)}$$

$$\text{Total time for 30 units} = 13.38 \times 30 = 401.4 \text{ hours}$$

$$\text{Total time for 50 units} = 11.35 \times 50 = 567.5 \text{ hours}$$

Time taken for 20 units from 31 to 50 units $(567.5 - 401.4) = 166.1$ hours

$$\begin{aligned} \text{(ii) Man hours} &= 10 \times 8 \times 5 \times 4 &= 1600 \\ \text{(-) down time (25\% \times 1600)} &= \underline{400} \\ &= \underline{1200} \end{aligned}$$

Fixed Cost per hour = $6000/1200 = ₹ 5$

Computation of total cost for the initial order

Material (30×30)	= ₹ 900.0
Labour (401.4×6)	= ₹ 2408.4
Variable Overheads (0.5×401.4)	= ₹ 200.7
Fixed Overheads (5×401.4)	= ₹ 2007.0
Total Cost	= ₹ 5516.1

Illustration 4

A firm received an order to make and supply eight units of standard product which involves intricate labour operations. The first unit was made in 10 hours. It is understood that this type of operation is subject to an 80% learning rate. The workers are getting wages at the rate of ₹12 per hour.

- What is the total time and labour cost required to execute the above order?
- If a repeat order of 24 units is also received from the same customer, what is the labour cost necessary for the second order?

Solution:

80% Learning Curve results are given below:

Production (Units)	Cumulative Average Time (hours)	Total Time (hours)
1	10	10
2	8	16
4	6.4	25.6
8	5.12	40.96
16	4.096	65.54
32	3.2768	104.86

Labour time required for first eight units = 40.96 hours

Labour cost required for 8 units = 40.96 hours \times ₹ 12/hr = ₹ 491.52

Labour time for 32 units = 104.86 hours

Labour time for first eight units = 40.96 hours

Labour time required for 2nd order of 24 units = $104.86 - 40.96 = 63.90$ hours

Labour cost for the 2nd order of 24 units = 63.90 hours \times ₹12/hr = ₹ 766.80

Illustration 5

A manager wants to determine an appropriate learning percentage for a new type of work his company will undertake. He has obtained unit times for the initial six repetitions which are given below.

Unit	1	2	3	4	5	6
Completion time (Minutes.)	46	39	35	33	32	30

Determine –

- The learning percentage.
- Estimate the average time if a total of 30 units are planned.
- After producing how many units, will completion time less than 15 Minutes per unit be achieved?

Solution:

- As per the concept of doubling of Learning Curve theory we have,

Completion time of 2nd Unit = Completion time of 1st Unit × Learning percentage or, $39 = 46 \times LP$
or, $LP = 0.8478$

Completion time of 4th Unit = Completion time of 2nd Unit × Learning percentage or, $33 = 39 \times LP$
or, $LP = 0.8461$

So the Learning Percentage (LP) can be taken as 0.85 (Approx.) i.e. 85%

- $y_x = aX^b$ where $a =$ Completion Time of the 1st Unit = 46 Mins, $X =$ Cumulative no. of units produced = 30
 $b =$ Learning Index = $[\log(\text{learning percentage})] / \log 2 = \log(0.85) / \log 2 = -0.2344$ & $y_x = y_{30} = ?$

So average completion time of 30 units = $y_{30} = 46.(30^{-0.2344}) = 20.73$ Mins.

- Using the above formula we have, $15 = 46(X^{-0.2344})$

$$\text{or, } \log 15 = \log 46 - 0.2344 \log X$$

$$\text{or, } \log X = 2.0762 \quad \text{or, } X = 10^{2.0762} = 119.179$$

Average completion time will be less than 15 minutes if 120 units are produced.

Illustration 6

XYZ Co. does not have any costing department. You need to help them out in finding the total cost for a contract of a single order of 600 units for one of their products.

The average unit variable cost data for an initial batch of 200 units are as follows –

- Direct Material – 15 m² @ ₹ 80 per m²
- Direct Labour –
8 Hours in Department 1 @ ₹ 75 per hour and 100 Hours in Department 2 @ ₹ 100 per hour
- Variable Overhead – 25% of Direct Labour

Labour times in Departments 1 and 2 are expected to follow 80% and 70% Learning Curves respectively.

Solution:

From Learning Curve Theory, we have $y = aX^b$

where y = Cumulative average direct labour hours per unit

a = Direct labour hours for the first unit

X = Cumulative number of units

b = Learning Index

As this is a case of Batch Production, we define 'a' as the time for production of the first batch of 200 units

For Department 1, we have $a = 200 \times 8 = 1600$ hours and Department 2, $a = 200 \times 100 = 20000$ hours

Department 1 has Learning efficiency of 80%. So $b = \log 0.8 / \log 2 = -0.322$

Department 2 has Learning efficiency of 70%. So $b = \log 0.7 / \log 2 = -0.515$

So formula for Department 1 is $y = 1600(X^{-0.322})$ and for Department 2, $y = 20000(X^{-0.515})$

Calculations

Direct Material Cost = $(15 \text{ m}^2 \times ₹80 \text{ per m}^2)$ per unit $\times 600$ units = $(₹1200) \times 600 = ₹ 7,20,000$

Direct Labour Cost –

As this is a case of Batch Production and 1st batch is of 200 units, the order of 600 units will be equal to 3 batches.

For Department 1, Cumulative average time to produce 600 units = $y = 1600(3^{-0.322}) = 1123.277$ hours per batch

Total time required for 3 batches = $1123.277 \times 3 = 3369.83$ hours

So the Labour Cost is $3369.83 \times ₹75$ per hour = ₹2,52,737.25

For Department 2, Cumulative average time to produce 600 units = $y = 20000(3^{-0.515}) = 11358.279$ hours per batch

Total time required for 3 batches = $11,358.279 \times 3 = 34,074.84$ hours

So the Labour Cost is $34,074.84 \times ₹ 100$ per hour = ₹ 3407484

Total Direct Labour Cost = ₹ $(2,52,737.25 + 34,07,484) = ₹ 36,60,221.25$

Variable Overhead – 25% of Direct Labour Cost = $0.25 \times 36,60,221.25 = ₹ 9,15,055.31$

Total Cost for 600 units is – ₹ $(7,20,000 + 3660221.25 + 915055.31) = ₹ 52,95,276$

Illustration 7

An automobile manufacturer is conducting a product recall after it was discovered that a possible defect in the braking mechanism could cause loss of braking in certain cars. The recall covers a span of three model years. The company sent out letters to the car owners promising to repair the defect at no cost at any dealership.

The company's policy is to pay the dealer a fixed amount for each repair. The repair is somewhat complicated and the company expects learning to be a factor. In order to set a reasonable rate for repairs, the company conducted a number of repairs in house. Thereafter it was decided that the appropriate pay is ₹7000 per repair based on a flat hourly rate of ₹1700 and a learning percentage of 90.

Shortly after the dealers began making the repairs, the company received complaint from several dealers that they were encountering resistance from the workers who felt the flat rate was way too low and were threatening to refuse the work on those jobs. One of the dealers collected data on task times and sent it to the company given as follows

1. Three mechanics each completed two repairs.
2. Average time for the first unit was 9.6 hours and for the second unit was 7.2 hours.

The dealer has suggested a rate of ₹8500 per repair. You have been asked to investigate the situation and prepare a report.

1. Prepare a list of questions that you will need to have answered in order to analyze the situation.
2. Comment on the information provided in the case.
3. What preliminary thoughts do you have on solutions to the points you have raised?

Solution :

1. The list of questions for analyzing the situation could be as follows
 - ⊙ Is the learning percentage considered by the manufacturer correct?
 - ⊙ What time per repair has been allocated by the manufacturer?
 - ⊙ After how many repairs this time is achievable?
 - ⊙ What is the total number of repair jobs?
 - ⊙ Is the demand of the workers for higher rate justified?
 - ⊙ Should the rate suggested by the dealer be acceptable?
2. Information provided in the case is somewhat incomplete. Though it has been mentioned that the cars manufactured within a span of 3 model years are recalled but no numerical figure about the total number of cars recalled is clearly mentioned. In absence of this figure, it is not possible for anyone to understand exactly after how many repairs the rate suggested by the manufacturer becomes feasible. Moreover due to the very nature of learning phenomenon, the time required to complete the initial jobs of repair are bound to be more resulting in demand for higher rate by the workers.
3. Solutions to the points raised in (1) above are as follows
 - ⊙ From the dealer's data of repair of 1st and 2nd cars we find the same 9.6 and 7.2 hours respectively. That means the Learning percentage is $7.2/9.6 = 0.75$ or $75\% < 90\%$ Hence the actual learning percentage is less than that considered by the manufacturer.
 - ⊙ The manufacturer's rate per repair is ₹7000 @ ₹1700 per hour. Thus, time allocated per repair is $7000 / 1700 = 4.12$ hours
 - ⊙ Considering the dealer's figure of 9.6 hours for the 1st job to be correct the calculations for finding the number of repairs required to achieve 4.12 hours per repair is done as below.

$$Y = aX^b$$

where Y = Average time to complete X number of jobs = 4.12 hours,

a = Time to complete the 1st job = 9.6 hours and

b = Learning Index = $\log(\text{learning percentage}) / \log 2 = \log(0.75) / \log 2 = -0.415$

$$4.12 = 9.6 \cdot X^{-0.416} \text{ Or, } X^{-0.415} = 4.12 / 9.6$$

$$\text{Or, } X^{-0.415} = 0.429 \quad \text{Or, } -0.415 \log X = \log(0.429) \quad \text{Or, } \log X = 0.886 \quad \text{Or, } X = 10^{0.886} = 7.7$$

Thus, the time considered by the manufacturer is achievable in 8th repair

- ⊙ Total number of repair jobs is not clearly spelt here, but it can be well assumed that the same would definitely be a few thousands.
- ⊙ Demand of the workers is not justified because after doing only 8 repairs the rate suggested by the manufacturer is achievable and thereafter the actual labour hours will be further reduced causing the workers to earn good amount because whatever hours they take the amount of ₹7000 they will definitely get.
- ⊙ As explained above the fixed rate of ₹7000 is quite a good rate and hence the same does not need a higher revision.

EXERCISE**A. Theoretical Questions:****⊙ Multiple Choice Questions**

1. A Learning Curve describes
 - (a) The increase in number of units produced per unit time as the total number of units produced increases
 - (b) The rate at which an organisation acquires new information.
 - (c) The amount of production time per unit as the total number of units produced increases.
 - (d) The increase in production time as the total number of units produced increases.
2. Limitations of the Learning Curve approach include –
 - (a) Learning Curves must be redeveloped whenever the product or the production process is modified.
 - (b) Learning Curves are applicable when considering a highly automated process.
 - (c) Learning Curves are only valid when considering simple production process.
 - (d) Learning Curves are only valid when the total number of units produced is relatively small.
3. Which of the following statements about Learning Curve is incorrect?
 - (a) A change in the process disrupts the Learning Curve.
 - (b) The rate of learning varies depending on the quality of management.
 - (c) The Learning Curve can be disrupted by the change in personnel.
 - (d) Learning Curves show that the time saved in completing each subsequent unit increases.
4. Which of the following is not an application of Learning Curve?
 - (a) Learning Curves allow a manager to predict the time required for new employee orientation on company policies and procedures.
 - (b) Learning Curves permit a manager to prepare a work schedule.
 - (c) Learning Curves allow a manager to forecast the labour requirements while preparing a departmental employee budget.
 - (d) Learning Curves can be employed in supply chain negotiations.
5. The Learning Curve remains valid –
 - (a) When applied to different firms in the same industry.
 - (b) For product modification that will simplify the product assembly.
 - (c) As long as process revisions involve only the addition of automated machinery.
 - (d) As long as production volume increases and processes are consistent.
6. A 100% Learning Curve implies that –
 - (a) Organisational learning has taken place.
 - (b) The Learning Curve can be applied universally across an industry.
 - (c) No learning has been achieved.
 - (d) Direct labour time is reduced 100% for each doubling of production.

7. Which of the following statement is true?
 - (a) The Learning Curve displays the relationship between process time per unit and the cumulative number of units produced.
 - (b) Learning Curves are most easily developed for one off type orders.
 - (c) Learning Curves are based on the Normal distribution.
 - (d) The standard time for a process is determined from the Learning Curve when cumulative production is fifty units.

8. Learning Curve is also known as –
 - (a) Growth curve
 - (b) Production curve
 - (c) Exponential curve
 - (d) Experience curve

9. For organisations with wide variety of product range which of the following statement is correct?
 - (a) Different product will have different Learning Curves.
 - (b) Learning percentage for different product will be different.
 - (c) Both (a) and (b)
 - (d) None of the above

Answers:

1	2	3	4	5	6	7	8	9
c	b	d	d	d	c	a	d	c

⊙ **State True or False**

1. Learning Curves are based on the premise that employees and organisations get better at their tasks as they are repeated.
2. Learning Curves are based on the doubling of output.
3. Disruption of Learning Curve is possible if the members of a crew are changed.
4. Reduction rate of time to produce an item is higher for the curves with higher learning percentage.
5. Costs tend to decline more for standardized products and processes.
6. In Service sectors, the concept of Learning Curve is not applicable.
7. The steeper the Learning Curve, the faster is the decrease in time.
8. Time needed to complete a job is reduced by 90% when the learning percentage is 90.
9. Regardless of the industry, the Learning Curve remains same if the products or processes are same.
10. Lowering costs through a learning effect is not a natural consequence of the passage of time. These costs must be managed down.
11. If the learning percentage is 80 and the fourth unit took 16 hours to complete then the first unit should have taken 28 hours.

12. The equation $\log Y = \log a + b \log X$ is by no means related to the theory of Learning Curve.
13. For very large one off type jobs learning has a very minimal effect on its cost of production.
14. In case of Surgery, the effect of learning is most predominant.
15. Theory of Learning Curve is established on the assumption that no interruptions are there in the process of production.
16. Completion time of a product follows a G.P with common ratio same as learning percentage when the production process is repetitive in nature.
17. The process of learning is applicable to all the living beings of the world.
18. Consideration of Learning Curve is not important while deciding on the working capital requirement.
19. Force majeure situations like strike, lock out, shut down, natural calamities etc. have no effect on the Learning Curve.
20. Learning Curve has three phases, of which the last one is very steep.

Answers:

1	2	3	4	5	6	7	8	9	10
T	T	T	F	T	F	T	F	F	T
11	12	13	14	15	16	17	18	19	20
F	F	T	T	T	T	T	F	F	F

⊙ **Fill in the blanks**

1. When the concept of learning was discovered then its curve was named as ____ curve.
2. Firms leapfrog the experience curve by means of innovation and ____
3. The Learning Curve is exponential in nature. In a log scale it is represented by ____ line.
4. Experience curve was developed by ____ Group in the year 1960.
5. Frequent training of new entrants is required for the industries with high ____ of employees.
6. ____ of the Learning Curve depends on the method of production.
7. Knowledge of Learning Curve assists in planning ____ of materials.
8. Learning Curve shows that the crew size need not be ____ at the same rate of increment of production.
9. Sensitive ____ bargaining with the union is possible when a clear knowledge of learning rate is there.
10. In the formula $Y = aX^b$ the symbol b stands for ____
11. As per the theory of Learning Curve the time required to produce an item follows a ____ with each doubling of production.
12. 80% Learning Curve shows a ____ reduction in average labour cost for every doubling of production.
13. Any Learning Curve shows ____ decrease in average production time in the first phase.
14. Today's concept of Learning Curve is the result of a study by a German ____ named Ebbinghaus.

15. Learning effect is very predominant for _____ type task.
16. Studies show that there is no _____ accepted learning rate applicable anywhere & everywhere.

Answers:

1.	Forgetting	2.	Invention
3.	Straight	4.	Boston consulting
5.	Turnover	6.	Slope
7.	Inventory	8.	Increased
9.	Wage	10.	Learning Index
11.	Geometric Progression	12.	20%
13.	Steep	14.	Psychologist
15.	Repetitive	16.	Universally

⊙ Short essay type questions

1. Why should the learning percentage be higher in an Automatic process than in Manual process?
2. What would a learning of 95 percent mean?
3. If the learning phenomenon applies to all human activities, why the effect is not noticeable in short cycle production?
4. What are the phases of Learning Curve?
5. What is meant by Learning Curve ratio?
6. What do you understand by “Learning process”?

⊙ Essay type questions

1. What is Learning Curve and why it is important?
2. What are the limitations of Learning Curve?
3. Discuss about the various uses of Learning Curve.

B. Numerical Questions

⊙ Multiple Choice Questions

1. After studying the activities of the employees of one department of a large retail outlet, the manager has come out with a Learning Curve by computing the value of b using the relation $b = \log r / \log 3$ instead of the traditional way. Which statement below is best suited to this?
 - (a) If both the department under study and another department take 10 hours to complete a task for the first time then the time required for performing the task 30th time is lower for the department under study.
 - (b) If both departments have a learning rate of 90% and take 10 hours for performing a task for the first time then the time required by the department under study for performing the task 27th time is the same as

the time required by the other department performing the task 8th time, given that the other department uses the traditional way of calculating b.

- (c) This method is not valid since the denominator of the formula has $\log 3$ instead of $\log 2$.
 - (d) The employees of the department under study learn faster than the other employees.
2. How long will it take to produce the 4th unit with 80% learning if the first unit took 75 hours?
- (a) 48 hours
 - (b) 117 hours
 - (c) 60 hours
 - (d) None of the above
3. Bimal and Kamal are the two industrial workers engaged in doing a similar job. They have different learning rates of 80% and 90% respectively. Times taken to complete their first jobs are respectively 12 and 8 hours. If both continue with the same learning rate then after how many units Bimal will be faster than Kamal?
- (a) 5th unit
 - (b) 7th unit
 - (c) 11th unit
 - (d) 19th unit
4. How long will it take to produce the fifth unit with 85% learning rate, if the third unit took 13 hours?
- (a) 10.3 hours
 - (b) 10.0 hours
 - (c) 11.4 hours
 - (d) 11.5 hours
5. When 24 hours is required to produce a condenser of a particular type then the time required to produce the 16th unit with 85% Learning Curve is –
- (a) Between 9 and 10 hours
 - (b) Between 12 and 14 hours
 - (c) Between 15 and 17 hours
 - (d) Between 18 and 20 hours
6. A diesel engine manufacturing company has an order of 4 large engines. A crew of 16 members took 4000 hours to assemble the first engine. If 80% Learning Curve is used then what will be the labour cost of the fourth engine, assuming average labour rate to be ₹ 180 per hour?
- (a) Between ₹ 4 to 4.5 Lakhs
 - (b) Between ₹ 3.5 to 4 Lakhs
 - (c) Between ₹ 3 to 3.5 Lakhs
 - (d) Between ₹ 4.5 to 5 Lakhs

7. A builder requires 4000 hours to complete his first house. The experienced team of workers has a learning rate of 80%. How much time will be required to complete the 10th house?
 - (a) Between 1900 and 2000 hours
 - (b) Between 2000 and 2100 hours
 - (c) Between 2100 and 2200 hours
 - (d) Between 2200 and 2300 hours

8. Using the data of the above question what is the total time required to complete the first 4 houses?
 - (a) Between 4000 and 7000 hours
 - (b) Between 7000 and 9000 hours
 - (c) Between 9000 and 11000 hours
 - (d) Between 11000 and 13000 hours

9. For a 90% Learning Curve which one of the following statement is correct?
 - (a) Production time of each unit is 90% of the production time of the preceding unit.
 - (b) Direct labour time is reduced by 10% each time the production volume doubles.
 - (c) The learning process causes a 90% saving in the material content of the product.
 - (d) Only 10% defectives are produced during the process of learning.

10. To determine the Learning Curve for an assembly activity in a factory, time study is carried out with the help of a stop watch which shows a minute in place of an actual time of 66 seconds. Which of the following statement suits best to describe this situation?
 - (a) The learning rate determined with this reading is 10% lower than the actual.
 - (b) The learning rate determined with this reading is 10% higher than the actual.
 - (c) A schedule to produce 50 units based on these calculations will have 10% excess time built into it.
 - (d) A schedule to produce 50 units based on these calculations will have 10% less time built into it.

11. A manufacturing unit has 95% Learning Curve. To bag a huge order of a special item, the company made a prototype of the same. It took 25 hours to complete. As per the standard norms laid by the management of the company, the learning period will be over when the per unit production time is reduced by 20%. To achieve this norm for this special item, the production has to run continuously for
 - (a) 736 hours
 - (b) 572 hours
 - (c) 428 hours
 - (d) 199 hours

Answer

1	2	3	4	5	6	7	8	9	10	11
b	a	c	d	b	d	a	d	b	c	c

⊙ **Comprehensive Numerical Problems**

1. A tool manufacturing company wants to sell 49 units to a customer. Calculate the minimum learning rate if the 49th unit cannot take more than 5.6 hours. Given:- 1st unit takes 16 hours
2. Assume that the first unit of a very special product takes 15 labour hours to produce and the Learning rate is 74%. How many units should at least be produced to reduce the time per unit to 6 hours or less?
3. A lot of 20 units is to be produced. Labour cost is ₹ 150 per hour. Set up cost is ₹ 3500 and Material cost is ₹ 1500 per unit. The learning percentage is expected to be 90. Overhead is charged at the rate of 50% of total Material, Labour and Set Up cost. Determine the average unit cost for the lot when the first unit took 5 hours to complete.
4. The organisation to which you belong is a regular supplier of Indian Railways. Submission of bids against various tenders is a regular affair of the firm. As a part of the organisation's Costing Department your job is to help the Department who are dealing with these bids by calculating the costs.

Recently there is a limited tender for supply of 1600 pieces of a product which is somewhat new to your organisation. Indian Railways want supply of batches of 100 at a time. It has been estimated that the average cost of the first batch of supply is ₹ 100 per piece. 90% Learning Curve applies to the cumulative labour cost on this contract.

- (a) On the basis of the information above prepare an estimate of labour cost for the full contract.
 - (b) As per the tender, a supplier will be awarded with another order of 800 pieces after the successful completion of supply of the first 1600. Find out the incremental labour cost for producing an additional 800 unit?
5. A firm has a training program for an operation. The progress of the trainees is carefully monitored. An established standard requires a trainee to be able to complete the sixth repetition of the operation in 6 hours or less. Those who are unable to do this are assigned to other jobs. Currently, three trainees have each completed two repetitions: Trainee Ajay had times of 9 hours for the first and 8 hours for the second repetition; Trainee Badal had times of 10 hours and 8 hours for the first and the second repetitions and trainee Chanchal had times of 12 and 9 hours. Which trainee or trainees will achieve the standard? Explain with reasons.
 6. The vendor of an automobile company is approached by the principals to build a special component for them. Based on the previous experience of the similar components, the following data have been made ready by the Costing Department of the vendor.

Breakeven quantity = 24 units, Time for producing the 24th unit = 161 hours, Learning rate = 80%

Direct labour cost = ₹ 120 per hour

- (a) Based on the above data what quote should be given to the principals for the direct labour cost for an order of 30 units?
- (b) The quotation given as per above is accepted by the principals and a green signal is given to the vendor to start production. Although the first few units took longer than the estimated labour hours but thereafter it has started to stabilize. However the customer has made some changes in design after the production has started. They want the change to be implemented after the 15th unit. As per the estimation, the design change will cause 30 hours of new work for the 24th unit when the Learning Curve remains same. In fact the new work replaces the same amount of work of 30 hours for the 24th unit. What should be the revised quotation, if any, for the order of 30 units?

Answers:

1. Minimum learning rate = 83%
2. At least 9 units should be produced.
3. Average unit cost = ₹ 3226
4. (a) Total cost of 1600 pieces = ₹ 104976,
(b) Incremental cost for the additional 800 pieces = ₹ 43078
[Hints : Additional 800 pieces after 1600 means a total of 2400 pieces which means 24 batches of 100 each. Also 2400 pieces is not the double of 1600. So the formula to be applied here is $Y = aX^b$ with $a = ₹ 100$, $X = 24$ and $b = \log (0.90) / \log 2$]
5. Both Badal and Chanchal will achieve the standard because for them the time to complete the 6th repetition are respectively 5.6 hours and 5.7 hours i.e. less than 6 hours, but for Ajay, the time is 6.64 hours.
6. (a) Quote for the direct labour cost for an order of 30 units = ₹ 539421
(b) Revised quote for the direct labour cost to make 30 units = ₹ 583000
[Hints : Due to design change there will be new learning effect from 16th unit onwards]

References:

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- www.onlinelibrary.com/doi/abs/10.1111/j.1540-5915.1979.tb00026.x